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A statistical framework for strategic decision making with AHP: Probability assessment and Bayesian revision[☆]

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Abstract

A probability assessment framework is outlined for an organizational decision involving a conditioning event (CE). The decision may, for example, involve a new-product launch (strategic decision) dependent on the outcome of market research (CE). The framework illustrates how Bayesian revision could be employed as related “news” arrives intermittently to revise current probabilities prior to decision implementation. A unique contribution of this paper is its utilization of the analytic hierarchy process to ascertain a set of consistent and coherent probabilities for the event/sample spaces at all stages of the decision process. © 2007 Elsevier Ltd. All rights reserved.

Keywords: Strategic decision making; Probability assessment; Analytic hierarchy process; Bayesian revision

1. Introduction

Analytical strategic decision making involves calculation of the expected value (or utility) of different options available to the corporation or decision maker. For example, launch, postpone or abandon a hostile takeover. The decision may also require the evaluation of commissioning more research on the problem and what cost should be incurred in the light of the expected value of the information likely to be gathered. These calculations involve probability assessment over the various outcomes and the possible updating of these probabilities.

The analytical hierarchy process (AHP) [1,2] is an established methodology for decision making as illustrated in recent publications [3–6]. AHP methodology

uses properties of reciprocal matrices to achieve consistency in pairwise judgments leading to a cardinal ranking of actions, objectives, attributes and criteria relevant to the decision situation. These matrix properties have been exploited in areas not directly related to AHP decision making such as foreign exchange markets as in forecasting exchange-rate movements [7] or investigating possibilities for profitable arbitrage [8].

In this paper, the properties of reciprocal matrices are exploited to achieve consistency in a probability distribution over a set of exhaustive and mutually exclusive events. The derived probabilities are envisaged as being used in conjunction with decision-making methodologies such as maximization of expected utility and AHP. First, AHP procedures are employed to derive prior distributions relevant to the decision situation. Subsequently, methodologies are outlined showing how Bayesian revision can be employed to update these prior probabilities in the light of randomly arriving “news” events also relevant to the decision situation. Simple

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examples illustrate how probabilities could be revised and how this could lead to a change in the optimal action.

2. Statistical framework for strategic decision making

This paper outlines the generic problem using a strategic decision (SD) with possible outcomes of success, failure and neutral (neither success nor failure) with sample space $\{S, F, N\}$. Thus, $P(S)$ could define the probability of a successful launch of a new product. If a complete analysis results in a $P(S)$ that is “too low”, the resulting decision would almost certainly be to abandon the launch. The details of the results for specific SDs such as a new-product launch, merger or license negotiations, sale of a non-core division are not discussed here but would involve established decision-theory criteria such as choosing an option that optimizes the expectation of some payoff function. This paper focuses on the probabilities that would be needed in the expectation calculus and, in particular, how the decision maker can take advantage of AHP procedures to translate initial vague relative-likelihood statements into numerically consistent and coherent probabilities.

In this analysis, the SD is assumed to be influenced by a conditioning event (CE) with possible outcomes good, indifferent or bad, hence $\{G, I, B\}$. For example,

market research on the new product may result in a good outcome raising $P(S)$, the posterior probability of success, thereby favoring the launch of the new product. Even before the result of this market research becomes available, “news” on similar or competing products could make possible revision of the probabilities $P(G)$, $P(I)$ and $P(B)$. Any such revision could raise or lower the value of revisiting the design of the current market-research task. We will suppose that G increases the prospects for success and B increases the prospects for failure. That is, the G and B outcomes for the CE are defined relative to S and F for the SD.

A good result for the CE will not necessarily reduce $P(F)$ to zero for the SD, although this probability will presumably be lower than it was prior to the good result. The first task is to assess the probabilities for the joint events in Table 1. This will lead eventually to $P_0(S), \dots, P_0(B)$ or more generally $P_t(\cdot)$ where time $t=0, 1, \dots, k, \dots, T$. Here, $t=0$ is the time of the first probability assessment, but probabilities could be revised through time as “news” becomes available, say in time period k . Even without a specific “news” event, the information set [9, p. 212] or knowledge base [10] used to derive the probabilities at time t could change from preceding periods. For example, previous “news” events could be re-assessed and/or reinterpreted and probabilities could be changed as a result. The outcome for the CE will be known during or prior to period T . The SD must be made at the end of period T . The planning for a major SD cannot always wait until the outcome of the CE is known before the end of period T , for example, arranging finance for a possible takeover. The statistical framework outlined here allows for these situations.

Some examples of SDs and associated CEs are shown in Table 2. These examples show some specific CEs that will influence the outcome of the SD. There is, however, general news arriving continually that will almost certainly impact the probability assessments for the

Table 1
Sample space of nine exhaustive and mutually exclusive events

Strategic decision	Conditioning event		
	Good	Indifferent	Bad
Success	$S \cap G$	$S \cap I$	$S \cap B$
Neutral	$N \cap G$	$N \cap I$	$N \cap B$
Failure	$F \cap G$	$F \cap I$	$F \cap B$

Table 2
Examples of a strategic decision with associated conditioning events

Strategic decision with sample space $\{S, N, F\}$	Conditioning event with sample space $\{G, I, B\}$
Launch, license new product, drug, etc.	Market-research outcome Clinical trial outcome Litigation outcome (e.g. patent dispute)
Takeover, merger, sale of unit or division	Earnings report and updates Antitrust decision Competitor's move
IPO, equity or debt issue	Stock market level Market climate or sentiment Results of similar issues by rivals

relevant events. These include announcements on GDP growth, inflation and employment as well as Central Bank or Federal Reserve pronouncements on interest rates, inflation, government economic policy, etc. Technological developments, election results, trade talks, world commodity prices and exchange-rate movements will also impact corporate and other strategic decision making. Whatever probability assessments have been derived with previous information could now be updated in the light of this “news”, which is arriving all the time. This allows room for utilizing Bayesian revision to update the prior or current probabilities in the light of recent “news” arrivals.

3. Prior probability assessments

Probability assessment of events (by individuals or groups) is a critical aspect of decision analysis leading to, for example, maximization of subjective expected utility as the criterion for decision selection [11]. The qualifier “subjective” here means that the so-called objective probabilities (e.g. from a frequency distribution) are typically not available for the events that have an impact on the decision situation. Of course, if they are available, they may be used directly or in forming a prior distribution for the decision maker. Since the relevant events will typically be unique to the decision situation, subjective judgments are usually required to determine the relative likelihood or probability of each event as in $P_i(S)$. In making assessments, probability axioms must be satisfied in order to prevent a Dutch-book outcome that could ensure a certain loss in a postulated gamble for the person making the assessments [12]. Reference lotteries, reason generation, scenario construction and prospective hindsight are some of the traditional techniques used to help decision makers determine their subjective probabilities of events [13].

Using AHP in subjective probability assessment has not been widespread to date. The first reference to AHP methodology for this purpose appears to be Yager [14]. His approach is very similar to that proposed here. More recently its potential usefulness was outlined by Hughes [15] and utilized in a medical situation by Monti and Carenini [16]. The AHP approach seems especially valuable in real-world situations where events generically described as good, indifferent and bad cannot be tractably modelled with traditional stochastic processes such as the normal (a real-numbered random variable), binomial (0, 1 or binary random variable), etc. Real-world events can be complex with multiple attributes all of which may not be quantifiable numerically using standard stochastic processes such as the normal,

binomial, etc. Saaty [2, p. 10] has noted the similarity of a derived AHP scale with a probability distribution:

The fundamental scale of the AHP is a scale of absolute numbers used to answer the basic question in all pairwise comparisons: how many times more dominant is one element than the other with respect to a certain criterion or attribute? ... The derived scale is like probabilities in not having a unit or an absolute zero.

In the present case, probability assessments on all nine joint events in Table 1 are required. We use the AHP with pairwise comparisons on the relative likelihood of all events. For example, is $F \cap B$ z times more likely than $S \cap G$ so that $S \cap G$ is $1/z$ as likely as $F \cap B$? In this case a 9×9 table shows the relative likelihoods for such pairwise judgments. It is suggested that decision makers use an approximate ranking of events from most likely to least likely before evaluating the relative likelihoods. Illustrative judgments are summarized in Table 3.

In Table 3 we see that $F \cap B$ is judged to be the most likely joint event and $S \cap B$ the least likely joint event. Looking at the ranking in this illustration, failure or neutral generally seems more likely than success excepting the case where a good result is the outcome of the CE. The italicized values are the reciprocals of the judgments in the upper right triangle where for an $m \times m$ matrix $m(m - 1)/2$ judgments are required ($9(8)/2$ or 36 in this case).

The AHP derivation of the resulting probability distribution is briefly outlined by taking the ranking in Table 3 as defining events 1–9. Accordingly, $F \cap B$ is event 1 and $S \cap B$ is event 9 with p_i the probability of event $i = 1, 2, \dots, 9$. In this context, the 6 in row 1 of Table 3 is the direct assessment of p_1/p_9 . In general, the assessed likelihoods or elements in Table 3 form the likelihood matrix L_{ij} with typical element p_i/p_j . This means the main diagonal elements of L are all unity and reciprocals p_j/p_i will comprise elements below the main diagonal. Using AHP methodology, the system of likelihood judgments summarized in matrix L with constituent probabilities $p_i, i = 1, 2, \dots, 9$, contained in a column vector p can be written as

$$(L - \lambda_{\max} I)p = 0.$$

Starting with the likelihood matrix L and deriving its maximum eigenvalue λ_{\max} enable us to solve the homogeneous system above determining the associated eigenvector which can then be normalized so its elements sum to unity. The resulting $p = (p_1, p_2, \dots, p_9)'$ will be the probability distribution we seek over all nine

Table 3
Likelihood judgments over events ranked from most to least likely

	$P(F \cap B)$	$P(N \cap B)$	$P(F \cap I)$	$P(N \cap I)$	$P(S \cap G)$	$P(N \cap G)$	$P(S \cap I)$	$P(F \cap G)$	$P(S \cap B)$
$P(F \cap B)$	1	2	2	3	3	4	4	5	6
$P(N \cap B)$	0.5	1	1	2	3	4	4	5	5
$P(F \cap I)$	0.5	1	1	2	2	3	3	4	5
$P(N \cap I)$	1/3	0.5	0.5	1	2	2	4	4	5
$P(S \cap G)$	1/3	1/3	0.5	0.5	1	2	2	3	3
$P(N \cap G)$	0.25	0.25	1/3	0.5	0.5	1	2	2	2
$P(S \cap I)$	0.25	0.25	1/3	0.25	0.5	0.5	1	3	3
$P(F \cap G)$	0.2	0.2	0.25	0.25	1/3	0.5	1/3	1	3
$P(S \cap B)$	1/6	0.2	0.2	0.2	1/3	0.5	1/3	1/3	1

Table 4
Geometric mean probability derivation for judgments in Table 3

	$P(F \cap B)$	$P(N \cap B)$	$P(F \cap I)$	$P(N \cap I)$	$P(S \cap G)$	$P(N \cap G)$	$P(S \cap I)$	$P(F \cap G)$	$P(S \cap B)$	Geometric mean	Normalized probabilities
$P(F \cap B)$	1.000	2.000	2.000	3.000	3.000	4.000	4.000	5.000	6.000	2.956911	0.255780
$P(N \cap B)$	0.500	1.000	1.000	2.000	3.000	4.000	4.000	5.000	5.000	2.198524	0.190178
$P(F \cap I)$	0.500	1.000	1.000	2.000	2.000	3.000	3.000	4.000	5.000	1.923241	0.166365
$P(N \cap I)$	0.333	0.500	0.500	1.000	2.000	2.000	4.000	4.000	5.000	1.440260	0.124586
$P(S \cap G)$	0.333	0.333	0.500	0.500	1.000	2.000	2.000	3.000	3.000	1.000000	0.086502
$P(N \cap G)$	0.250	0.250	0.333	0.500	0.500	1.000	2.000	2.000	2.000	0.702495	0.060768
$P(S \cap I)$	0.250	0.250	0.333	0.250	0.500	0.500	1.000	3.000	3.000	0.610142	0.052779
$P(F \cap G)$	0.200	0.200	0.250	0.250	0.333	0.500	0.333	1.000	3.000	0.421135	0.036429
$P(S \cap B)$	0.167	0.200	0.200	0.200	0.333	0.500	0.333	0.333	1.000	0.307653	0.026613
Column sums										11.560361	1.000000

events in Table 3. Saaty [1,2] provides a complete analysis of AHP methodology.

Software packages such as MATLAB can be used for the maximum eigenvalue (eigenvector) method for probability calculation. An alternative procedure derives the geometric mean of each row in Table 3 and then normalizes the results to sum to one [17]. This is routinely accomplished using any standard spreadsheet package as shown in Table 4.

The probability for the first joint event $P(F \cap B)$ is derived as 2.956911/11.560361 with probabilities for all events shown in the last column of Table 4. Resulting probabilities for the judgments in Table 3 using both procedures are compared in Table 5.

Both procedures result in probabilities that are very close for each joint event. The maximum difference in absolute value is less than one-third of 1% (−0.0031). Budescu et al. [18, p. 77] compared both procedures and concluded that “there is a large consensus that in most cases the solutions are very similar to each other”. In what follows we use the probabilities derived using the maximum eigenvalue (eigenvector) procedure.

Table 5
Probability distributions for judgments in Table 3

Event	Probability distributions		Difference
	Eigenvector	Geometric mean	
$P(F \cap B)$	0.2557	0.2558	−0.0001
$P(N \cap B)$	0.1886	0.1902	−0.0016
$P(F \cap I)$	0.1632	0.1663	−0.0031
$P(N \cap I)$	0.1260	0.1246	0.0014
$P(S \cap G)$	0.0854	0.0865	−0.0011
$P(N \cap G)$	0.0608	0.0608	0.0000
$P(S \cap I)$	0.0552	0.0528	0.0024
$P(F \cap G)$	0.0380	0.0364	0.0016
$P(S \cap B)$	0.0271	0.0266	0.0005
Total	1.0000	1.0000	0.0000

As AHP theory documents, the multiple judgments made in these situations can result in some inconsistencies and the consistency ratio (CR) in this case at 0.04 is considered to be acceptable [2, p. 32]. If the consistency result was not acceptable (a CR exceeding 0.1), the decision maker should revise the judgments in Table 3.

Using probability axioms, we can now derive the $P_0(\cdot)$ probabilities as follows:

$$P_0(F) = P(F \cap G) + P(F \cap I) + P(F \cap B) = 0.4569,$$

$$P_0(N) = P(N \cap G) + P(N \cap I) + P(N \cap B) = 0.3754,$$

$$P_0(S) = 1 - P_0(F) - P_0(N) = 0.1677.$$

Similarly, for the CE we can derive

$$P_0(G) = 0.1842, \quad P_0(I) = 0.3444,$$

$$P_0(B) = 0.4714.$$

The likelihood judgments summarized in Table 3 currently do not seem to favor success for this SD and do not favor a good outcome for the CE which, if it did occur, would enhance the posterior likelihood of success.

There are alternative approaches to deriving the above prior probabilities. One possibility would be to use the AHP in a 3×3 evaluation for the CE. This would give $P_0(G)$, $P_0(I)$ and $P_0(B)$ directly. Similarly, the nine conditional probabilities could be derived in three separate 3×3 AHP evaluations. That is, evaluating the three relative likelihoods for $P(S|G)$, $P(N|G)$, $P(F|G)$, then the three probabilities conditional on I and finally the three probabilities conditional on B . The procedures required with this approach are essentially the same as presented in Tables 3–5 but on a smaller scale. For any organization continuously involved with SDs, the best way to proceed should arise naturally and quite possibly judgments may improve with practice. Smaller dimension judgment matrices (as in 3 versus 9) usually result in closer agreement between the probabilities derived using the eigenvalue and geometric mean procedures [17].

The above analysis assumed real-valued point estimate judgments of “more likely than” as summarized in Table 3. Corporations and organizations making SDs are almost certainly assigning teams of executives to complicated tasks raising the possibility that the team may not agree on a point estimate for a pairwise judgment such as 2 in Table 3. Rather, an interval judgment such as 1.5–3.5 could be the range of judgments held by team members. This raises the question as to the best point estimate to employ in this situation. Using optimization and simulation, Moreno-Jimenez and Vargas [19] outline a procedure for determining point estimate judgments from interval judgments. Hauser and Tadikamalla [20] extended this approach using simulations derived from a uniform distribution or a triangular distribution based on a modal value in the interval. This latter distribution would be appropriate where a

majority of the group favor one value intermediate to the extreme values favored by only a few. If, however, point estimates of individuals are evenly distributed over the interval, then a uniform distribution seems appropriate for determining the point estimate.

The advantage of the AHP approach to probability assessment over traditional methods is the capability to accommodate qualitative and quantitative factors in defining events and determining a resulting probability distribution. With only precisely specified quantitative factors involved in the judgments/measures (e.g. from frequency count data), a perfectly consistent probability distribution will be derived with λ_{\max} equal to the dimension of the likelihood matrix L [1,2]. If qualitative factors in whole or in part define the events, traditional procedures such as reference lotteries can still be used in deriving the necessary AHP inputs. Probability determination, however, could involve problems of interval judgments from teams and/or inconsistency of judgments/measures in the matrix L . A distorted probability distribution could be the result. Consequently, unless acceptable agreement and consistency can be achieved in the underlying judgments/measures, utilization of a distorted probability distribution may lead to incorrect strategic decision making with the AHP approach.

With a codified approach to a succession of SDs and associated probability assessments for important events, it would be expected that judgments/measures would become more “expert” (less undue optimism or pessimism) over time. Further, the debriefing for successful or unsuccessful SD initiatives would have an information audit trail that may help in replicating success and minimizing failure in future SDs.

4. Illustration of Bayesian revision

If “news” arrives related to the CE, Bayes theorem can be used to re-assess the likelihoods of the alternative states (G , I or B in this case). Here, “news” is given and we need to now determine the relative likelihoods of the three possibilities G , I or B in the light of this “news”. Given the “news” event, a pragmatic approach would be to first rank the states from least likely to most likely in light of this “news” and define the least likely as having likelihood 1. Subsequent relative likelihoods could be derived as “ x times more likely than” where x is at least 1 and non-decreasing with progression through to the most likely state. This is a natural extension of the method used to derive the pairwise likelihood judgments as in Table 3. Alternatively, likelihood evaluations can be made using traditional uncertainty measures such as probabilities, odds and percentages. As shown in

Table 6
Bayesian revision for CE likelihood after “news” of executive resigning

Outcome of CE (1)	Prior (2)	Likelihood of “news” for CE outcome (3)	Joint (4) = (2)*(3)	Posterior (5) = (4)/sum(4)
Bad	0.4714	1—“News” has the least likelihood of being consistent with bad compared with the other two alternatives	0.4714	0.1806
Indifferent	0.3444	3—Consistency of “news” with indifferent has triple the likelihood as that for bad	1.0332	0.3959
Good	0.1842	6—Consistency of “news” with good has twice the likelihood as that for indifferent	1.1052	0.4235
	1.0000		2.6098	1.0000

Table 6, any measurement unit for the likelihood judgments (column (3)) cancels out with the normalization in deriving the posterior distribution (column (5)).

For purposes of illustration, let us suppose the SD here relates to a takeover offer. The CE in this case is assumed to be the earnings report of the target firm, which is due shortly. The $P_0(B)$ value above suggests that the prior view is that this will be bad for the SD. That is, earnings of the target company are not currently seen as likely to be catastrophic and this could make the takeover more difficult and/or more expensive. Before the earnings report is due, however, news arrives that a major executive in the target firm has resigned. How can we incorporate this “news” into our analysis for a successful takeover? Some hypothetical judgments using the pragmatic approach outlined above are shown in Table 6.

Here, it is assumed that the executive is associated with a division or initiative regarded as crucial for the continued profitability of the target company. His/her resignation could signal a problem in this important division of the target company foreshadowing a good outcome for the CE of the potential acquirer. The likelihood judgments in Table 6 reflect this reasoning and underlie the resulting posterior probabilities which almost reverse the prior probabilities. Using P' for prior and P'' for posterior we see that P'' (good) at 0.4235 is just less than P' (bad) at 0.4714. Conversely, if the resignation was not directly related to the future earnings or performance of the target company (e.g. executive lured away to a better job in another company), the likelihoods in Table 6 (column (3)) may not favor good as much as is shown there. The key judgment is assessing the relative likelihood that the “news” has for consistency with a possible state of the world such as $\{G, I, B\}$ above.

In more complex situations, such as the joint events of Table 1, the multiple-judgment AHP approach summarized in Tables 3–5 could be employed to derive the

Bayesian likelihoods directly. The 1, 3 and 6 values in Table 6 (or their reciprocals) would then be one column (or row) in such a 3×3 matrix. A perfectly consistent judgment matrix (starting with the 1, 3 and 6 values in Table 6) would yield probabilities for G, I and B of 0.6, 0.3 and 0.1, respectively. For larger dimensions in a more complex situation, however, consistency of multiple-judgments may be more of a problem with revised assessments needed before acceptable consistency is achieved. Alternatively, likelihoods expressed using odds judgments of 9:1 against for B , 7:3 against for I and 3:2 on for G would yield the same probability distribution over the CE sample space. Using these probabilities would lead to the same posterior as derived in Table 6. The power of Bayesian revision is that it allows judgment to reflect the nuances of the “news” situation but the ultimate value of this capability depends on just how good this judgment is.

From the prior probability judgments in Table 5, conditional probabilities are derived as follows:

$$P(S|G) = 0.0854/0.1842 = 0.4636$$

$$\text{similarly } P(S|I) = 0.1603 \quad \text{and} \quad P(S|B) = 0.0575.$$

These gave the prior probability of success as

$$P'(S) = 0.0854 + 0.0552 + 0.0271 = 0.1677.$$

And now the posterior probability of success will be:

$$P''(S) = P(S|\text{“news”}) = 0.4636(0.4235) + 0.1603(0.3959) + 0.0575(0.1806) = 0.2702.$$

The posterior probabilities for success, etc. will be based in part on conditional probabilities assessed in the pre-news state. It could be argued that a better alternative would be to repeat the initial analysis of Table 3 in light of the new information since assessments of the joint probabilities could then change more appropriately. That is, the “news” event could have implications beyond the CE influencing the SD itself. If so,

the procedure outlined in and following Table 3 could be re-employed (or an equivalent alternative approach re-employed) and revised probabilities derived without using Bayesian revision at this stage. For this illustration, the executive resignation news has significantly increased the probability of success in the forthcoming SD. This could change the allocation of resources in pursuing the SD as well as the level of the offer (and financing) seen necessary to achieve the takeover.

5. Value of information changes due to a “news” event

Continuing with the above takeover example, we can now illustrate how the optimal act and value of information could change as a result of the executive resignation news. The payoffs in \$ millions suitably discounted over future years to reflect total estimated net wealth change for the action taken are assumed to be as summarized in Table 7.

The net wealth change attributable to abandoning the takeover (i.e. financial resources now employed elsewhere) shows a \$2 million gain irrespective of what would have happened had the takeover proceeded. A neutral outcome after the takeover shows the same return but all numbers in Table 7 are merely illustrative. For example, the abandon takeover payoffs could differ for each state if S , N and F also reflected changed economic prosperity in general leading to payoffs of say 3, 2 and 1, respectively, in the last row of Table 7. Table 8

outlines the resulting regret or opportunity loss table derived from Table 7 and shows the optimal acts under the prior and posterior, respectively, using the criterion of expected opportunity loss (EOL). This assumes that optimal decisions are made based on expected value and risk neutrality.

The asterisked EOL value indicates the lowest expected value of perfect information (EVPI) and the resulting optimal act under risk neutrality [21]. In this case the “news” event, which raised the probability of success from 0.1677 to 0.2702, changes the optimal act from abandon to launch takeover. It also raises the EVPI from \$1.1739 million to \$1.4916 million. Even though a prior optimal decision may be re-affirmed under the posterior, the value of research may be changed after the “news” event leading to a lesser or greater effort in this regard.

6. Conclusions

The eventual outcome of the CE prior to or during period T will reduce the sample space to three possible events $\{S, N, F\}$. Assuming the CE outcome is I , conditional probabilities $P(S|I)$, $P(N|I)$ and $P(F|I)$ incorporating all information to date would be used in determining the best SD option. Depending on the CE, traditional probability models could be used to generate these conditional probabilities. For example, S , N and F could be defined by various ranges of market share for a new product with the CE being market research designed to estimate possible market share for the new product. In this case, the final probabilities for S , N and F could be generated using a posterior beta distribution over market share. Alternatively, the AHP procedures illustrated here could be used where traditional stochastic models cannot be easily fitted to the decision situation. Finally, having more than one CE influencing the SD presents no particular problems (e.g. a sequence of CEs) although the complete sample space will now be expanded with increased computational requirements.

Table 7
Net wealth change in millions of dollars depending on action taken

Action taken	Outcome of strategic decision		
	Success	Neutral	Failure
Launch takeover	9	2	-2
Abandon takeover	2	2	2

Table 8
Regret or opportunity loss table in \$ millions for takeover decision

Action taken	Takeover outcome			EOL	
	Success	Neutral	Failure	Prior	Posterior
Launch takeover	0	0	4	1.8276	1.4916*
Abandon takeover	7	0	0	1.1739*	1.8914
Prior	0.1677	0.3754	0.4569		
Posterior	0.2702	0.3569	0.3729		

The procedures documented by AHP theory and illustrated here provide an ideal methodology for obtaining consistent probabilities at all stages of the decision process including Bayesian revision following real-world “news” events.

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