THINKING PROBABILISTICALLY REVISITED

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This brief demonstrates how a "ballpark" distribution may be altered to better reflect the decision-maker's (DM's) final judgments on the probabilities of possible events or scenarios.

The very simple procedure is reviewed showing how to quantify beliefs about possible events or scenarios when initially only rudimentary or tentative ideas about the relative likelihoods have been formulated. As a first step the possible events are ordered from least to most likely. Then a series of pairwise judgments for events X and Y are made by the DM qualitatively with suggested associated quantitative values as in:

- X and Y are equally likely (1.0)
- X is a little more likely than Y (1^+ – 1.25 or average $2.25^+/2$ or $1.13^+ \sim 1.15$)
- X is not quite twice as likely as Y (1.75 – $2^- < 2$ or average $1.88^- \sim 1.9$)
- X is between 2 and 3 times more likely than Y (2.5)

Of course, quantifying qualitative ideas accurately takes practice. One way is to average a range of values as above. Say you believe X is a "little more" likely than Y. But should "little more" be 10% or 40% more likely? Using an X/Y ratio of (1.1 + 1.4)/2 or 1.25 will get you to the "ballpark". Over time, exactly what a "little more" means to you can be refined more precisely.

The pairwise values used as below form the basis for deriving the "ballpark" probability distribution. They may only be vague, and the approximations above may be useful. They may be described as the "primary values."

To illustrate the methodology, take a situation with four events A, B, C and D and allow this ordering to be from least to most likely in the DM's view. Probabilities can be determined as in the following table, with A least likely (base value 1.0) and D most likely.

COMPUTATION OF PROBABILITIES FROM INITIAL PAIRWISE JUDGMENTS

Event	Pairwise Value	Compound Value	Probability	% Probability	ML % P(·)
A	1.00	1.00	1.00/7.33 = 0.136	14%	Base = 1.0
B	1.20	1.00 x 1.20 = 1.20	1.20/7.33 = 0.164	16%	16/14 = 1.14
C	1.90	1.20 x 1.90 = 2.28	2.28/7.33 = 0.311	31%	31/16 = 1.94
D	1.25	2.28 x 1.25 = 2.85	2.85/7.33 = 0.389	39%	39/31 = 1.26
TOTALS		**7.33**	**1.000**	**100%**	

After the base value 1.0, we have B a little more likely than A with a pairwise value of 1.2 for the B over A or B/A value. Event C is judged not quite twice as likely as B so the value 1.9 is assigned here for C/B. Finally, D is judged to be slightly more likely than C with the 1.25 value assigned as a result. Compound values are easily determined as in column 3 with the C/A value necessarily 2.28 and the D/A value slightly higher as dictated by the preceding pairwise values. Probabilities are then determined as in column 4 and percentage probabilities follow. Note that we use D/A as shorthand for the pairwise likelihood ratio for D over A with D/A = (B/A)(C/B)(D/C).

Faced with the resulting "ballpark" probability distribution above (columns 4 and 5), the DM could finalise his/her probabilities using the following (illustrative) arguments. The probabilities for A and B sum to 30% but this should be reduced to 25% in the DM's judgment. Hence use 10% and 15%

respectively with 1.5 the final pairwise value as opposed to the original 1.2. Raise the probability of the outcomes C and D collectively to 75% at 35% and 40% respectively. These judgments are made easier because the "ballpark" probabilities in columns 4 and 5 above are already available to the decision-maker. These final judgments result in pairwise values for C/B at 2.33 versus 1.9 originally and D/C at 1.14 versus 1.25 originally. There is nothing sacrosanct about the original pairwise values and exact matching is not expected of the final probability distribution as in columns 2 and 6 in the above table. The original pairwise values are a means to an end and not set in stone. Other information may come into play in deciding on the final distribution as the above illustrates. Note too that the final distribution could be determined to avoid any charge of spurious accuracy as in 32% (28%) rounded down (up) to 30% in the final distribution.